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ON A DYNAMIC CONTACT PROBLEM FOR A SINGLE ELECTRODE*

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The dynamic problem of surface-wave excitation by the main element of electrode transducers, a single electrode simulated by a strip stamp lying freely on the surface of a piezoelectric half-space, is considered. The vertical component of the displacement and the electrical potential is given in the contact region, while the surface outside this region has no electrical and mechanical loads. The boundary value problem of electroelasticity mentioned reduces to investigating a system of inhomogeneous Fredholm type integral equations of the first kind in the unknown normal stress and charge distribution density functions.

The regularization method for the system of integral equations obtained is based on constructing the factorization of the kernel matrix-function and enables the system of integral equations of the first kind to be reduced to a system of integral equations of the second kind with a completely continuous operator for which separation into finite-dimensional and small terms is effective. Solutions are obtained for this system, that describe the behaviour of the contact stresses and the charge distribution density on the electrode, as well as the displacement and potential wave fields on the free piezoelectric surface, with the assignment of the electrical and mechanical perturbations taken into account. The absolute values of the deviation of the excited wave phase velocity from the Rayleigh wave velocity are computed at given points on the ST-cut surface of a piezoelectric quartz crystal.

Approaches developed earlier for constructing approximate solutions of the problems of the excitation and interaction of surface waves with metallic electrodes are based, as a rule, on the assumption of the weightlessness of the electrodes without taking account of the influence of the mechanical perturbations and the nature of the contact with the medium /1-3/. At high frequencies as well as during examination of resonators these factors are of no little importance.

1. We introduce an $Ox_1x_2x_3$ coordinate system and we assume that the crystal occupies the domain $x_3 \leqslant 0$; x_1 is the wave propagation direction and the electrode dimensions along the x_2 axis are infinite.

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The wave fields excited by an electrode $[a_1, a_2]$ (a_1 is the origin and a_2 the termination of the electrode) lying simply supported on the surface of a piezoelectric field when electrical and mechanical loads are applied are described by a combined system of dynamic equations of motion and Maxwell's equations in the quasistatic approximation /3/ with mixed boundary conditions

$$\rho\frac{\partial^2 U_i}{\partial t^2}=\frac{\partial \sigma_{ij}}{\partial x_j},\quad \frac{\partial D_m}{\partial x_m}=0 \tag{1.1}$$

$$\sigma_{ij}=c_{ijkl}\frac{\partial U_k}{\partial x_l}-e_{nij}E_n,\quad D_m=e_{mkl}\frac{\partial U_k}{\partial x_l}+\varepsilon_{mn}E_n$$

$$E_n=-\frac{\partial \Psi}{\partial x_n},\quad i, j, k, l, m, n=1, 2, 3$$

$$\sigma_{13}=0,\ \sigma_{23}=0 \tag{1.2}$$

$$U_3=F_1(x_1, t),\ \Psi=F_2(t),\ x_3=0,\ x_1\in(a_1, a_2)$$

$$\sigma_{j3}=0,\ D_3=D_3^0(x_1, t),\ x_3=0,\ x_1\overline{\in}(a_1, a_2)$$

The steady-state harmonic oscillation mode is considered, namely,

$$U_i=u_i(x_1, x_3)e^{i\omega t},\quad \Psi=\psi(x_1, x_3)e^{i\omega t},\quad E_n=E_n(x_1, x_3)e^{i\omega t}$$

$$D_m=D_m(x_1, x_3)e^{i\omega t},\quad F_1=f_1(x_1)e^{i\omega t},\quad F_2=f_2e^{i\omega t}$$

Here U_i are displacement vector components, Ψ is the electric potential, E_n, D_m are the piezoelectric electric field and induction vector components, D_3^0 is the normal component of the electric induction vector of a vacuum, ω is the frequency of electrode vibration, c_{ijkl}, e_{mkl}, ε_{mn} are the elastic, piezoelectric, and permittivity vectors $f_1(x_1)$, f_2 are given amplitude values of the normal components of the displacement and the electric potential that has a constant value on the electrode surface. Without loss of generality, we set $f_1(x_1)=k\exp(i\eta x_1)$. For an electrode with a flat underside $\eta=0$.

Solving the dynamic contact problem (1.1) and (1.2), we find the displacement and potential wave fields on the free surface and we also study the behaviour of the contact stresses and the charge distribution density on the electrode. To do this we reduce the mixed problem (1.1) and (1.2) to a system of integral equations by first solving the auxiliary problem obtained from the mixed problem by assuming that the contact stress and the charge density on the electrode are known. The boundary conditions of the auxiliary problem are formulated as follows

$$\sigma_{13}=0,\ \sigma_{23}=0,\ \sigma_{33}=q_1(x_1),\ D_3(x_1)-D_3^0(x_1)=q_2(x_1),\ x_3=0 \tag{1.3}$$

$$q_1(x_1)=0,\ q_2(x_1)=0,\ x_1\overline{\in}(a_1, a_2)$$

In conformity with the physical principle of ultimate absorption /4/, the problem (1.1) and (1.2) posed above is reduced by the method of the Fourier integral transform using the solutions of the auxiliary problem (1.1) and (1.3), to a system of integral equations of the form

$$\int_{a_1}^{a_2}\mathbf{k}(x_1-\xi)\mathbf{q}(\xi)\,d\xi=\mathbf{f}(x_1),\quad x_1\in(a_1, a_2) \tag{1.4}$$

$$q=\{q_1(x_1), q_2(x_1)\},\ f=\{f_1(x_1), f_2\}$$

$$\mathbf{k}(x_1)=\frac{1}{2\pi}\int_\Gamma \mathbf{K}(u)e^{-iux_1}\,du,\quad \mathbf{K}(u)=\|K_{ij}(u)\|_{i,\,j=1,\,2}$$

The elements of the matrix-function of the kernel $K_{ij}(u)$ are awkward in form*,(*Ryzhkova T.V., On the vibration of an elastic half-plane and the phase velocities of surface acoustic waves. Dep. in VINITI June 15, 1984, No.3985-84, Rostov, 1984.) and their properties are investigated numerically using a computer.

Let us note the fundamental properties of the elements of $K_{ij}(u)$.

1°. On the real axis the functions $K_{ij}(u)$ have four points $\pm A_k$ $(k=1, 2)$, $A_2>A_1$, and two poles $\pm\zeta$.

2°. The functions $K_{ij}(u)$ are complex in the domain $A_{(1)}\leqslant|u|\leqslant A_2$, on the rest of the axis the $K_{ii}(u)$ are real, and $K_{ij}(u)$ $(i\neq j)$ have pure imaginary values. The equality $K_{12}(u)=-K_{21}(u)$ holds, the $K_{ii}(u)$ are even in u, and the $K_{ij}(u)$ $(i\neq j)$ are odd.

3°. As $|u|\to\infty$ the elements of the matrix function $K(u)$ have the following form apart from the factor $(1+O(|u|^{-1}))$:

$$K_{11}\approx A|u|^{-1},\ K_{22}\approx B|u|^{-1},\ K_{12}\approx iCu^{-1},\ A>B,\ B\gg|C| \tag{1.5}$$

In the representation (1.4) the contour Γ agrees with the real axis, deviating from it only by bypassing the positive singularities from above and the negative ones from below.

The slits drawn from the branch points to infinity and separating the single-valued branches of $K_{ij}(u)$ lie in the first and third quadrants /5/.

The theorems of the uniqueness and single-valued solvability of systems of integral equations similar to (1.4) are proved in /5/.

2. The theory for the solution of systems of integral equations given on segments by factorization of the matrix-functions of the kernel is outlined in /5/.

In the notation used in /5/, we obtain approximate formulas for the unknown vector-functions $\mathbf{q}(x_1)$, $\boldsymbol{\varphi}^{\pm}(x_1) = \{\varphi_1^{\pm}(x_1), \varphi_2^{\pm}(x_1)\}$ ($\varphi_1^{\pm}(x_1)$ are the amplitude values of the displacements along the x_1 axis, and $\varphi_2^{\pm}(x_1)$ are the amplitude values of the potential)

$$
\begin{aligned}
& a_1 < x_1 < a_2, \quad \mathbf{q}(x_1) = \frac{1}{2\pi}\int_{\Gamma} \mathbf{K}^{-1}(u)\,\mathbf{F}(u)\,e^{-iux_1}\,du - \\
& \quad i\sum_{k=1}^{n_1}\{\operatorname{Res}(\mathbf{N}_1^{-1}, c_k)\,\mathbf{X}_2^+(c_k)\exp(ic_k(a_2-x_1)) - \\
& \quad \operatorname{Res}(\mathbf{M}_+, b_k)\,\mathbf{X}_0^-(b_k)\exp(ib_k(a_1-x_1))\} \\
& x_1 > a_2, \quad \boldsymbol{\varphi}^+(x_1) = i\sum_{j=1}^{n_2}\operatorname{Res}(\mathbf{N}_+^{-1}, \zeta_j^-)\,\mathbf{X}_2^+(\zeta_j^-)\times \\
& \quad \exp(i\zeta_j^-(a_2-x_1)) - i\sum_{k=1}^{n_1}\mathbf{N}_+^{-1}(b_k)\,\mathbf{T}(b_k)\exp(ib_k(a_2-x_1)) \\
& x_1 < a_1, \quad \boldsymbol{\varphi}^{-1}(x_1) = -i\sum_{j=1}^{n_\circ}\operatorname{Res}(\mathbf{M}_-^{-1}, \zeta_j^+)\,\mathbf{X}_0^-(\zeta_j^+)\times \\
& \quad \exp(i\zeta_j^+(a_1-x_1)) + i\sum_{k=1}^{n_1}\mathbf{M}_-^{-1}(c_k)\,\mathbf{S}(c_k)\exp(ic_k(a_1-x_1)) \\
& \mathbf{K}(u) = \mathbf{M}_-^{-1}(u)\,\mathbf{M}_+(u) = \mathbf{N}_+^{-1}(u)\,\mathbf{N}_-(u) \\
& \mathbf{T}(b_k) = \operatorname{Res}(\mathbf{N}_+, b_k)\,\mathbf{M}_-^{-1}(b_k)\,\mathbf{X}_0^-(b_k)\exp(ib_k(a_2-a_1)) \\
& \mathbf{S}(c_k) = \operatorname{Res}(\mathbf{M}_-, c_k)\,\mathbf{N}_+^{-1}(c_k)\,\mathbf{X}_2^+(c_k)\exp(ic_k(a_2-a_1)), \quad \zeta_1^+ = \zeta, \quad \zeta_1^- = -\zeta
\end{aligned}
\tag{2.1}
$$

Here c_k ($c_k = -b_k$) are zeros of the determinant $\mathbf{K}(u)$ lying above the contour in the complex plane; factorization is performed with respect to the contour Γ; $\zeta_j^{\pm}$ ($j = 2, \ldots, n_2$) are parameters of the approximation of the matrix-function $\mathbf{K}(u)$ by a polynomial matrix, and $\operatorname{Res}(\mathbf{N}_+, b_k)$ is the residue of the matrix-function $\mathbf{N}_+(u)$ at the point b_k.

The amplitude functions of the contact stresses and charge distribution at the electrode edges have the following singularities

$$
\begin{aligned}
& \mathbf{q}(x_1) = \mathbf{r}(x_1-a_1)^{-1/2-\varepsilon}, \quad x_1 \to a_1+0, \quad \mathbf{q}(x_1) = \mathbf{r}(a_2-x_1)^{-1/2-\varepsilon}, \quad x_1 \to a_2-0 \\
& \varepsilon = \frac{i}{\pi}\operatorname{arth}\frac{C}{\sqrt{AB}}
\end{aligned}
\tag{2.2}
$$

(A, B, C are constants characterizing the asymptotic behaviour of the elements $K_{ij}(u)$ of (1.5)).

3. Numerical realization of the dynamic contact problem under consideration was performed using a computer. Results showed that the electrode dimensions and the nature of the applied load effect the amplitude characteristics of the contact stresses and the charge distribution denisty substantially. It is established that the oscillating nature of the contact stresses and the charge distribution density increase with electrode length; the amplitude charge distribution functions take the highest value on electrodes of small size. On superimposing both kinds of electrical and mechanical loads the mechanical load exerts a governing influence on the amplitude dependences on the contact stresses and the electrical load exerts a similar influence on the charge distribution characteristics.

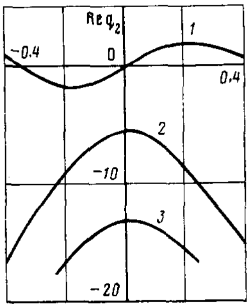
The figure shows graphs of the real part of the amplitude charge distribution function $\operatorname{Re} q_2(x_1)$ for different electrode dimensions, obtained when electrical and mechanical loads are applied. The x_1 coordinate is expressed in wavelengths $\lambda = 2\pi v_0/\omega$, where $v_0 = 3158$ m/s is the velocity of the Rayleigh length for the ST-cut of a quartz piezoelectric crystal, and $\omega = 10^5$ Hz. Curve *1* is obtained for the case when the normal displacement component is given in the contact domain ($k = 1$, $\eta = 0$, $f_2 = 0$), curves *2* and *3* for an electrode of width λ correspond to the cases when only the potential ($f_1 = 0$, $f_2 = 1$) is given on the surface of an electrode of width λ and 0.5 λ

The amplitude dependences of the contact stresses and the charge distribution density are obtained in dimensionless form. Consequently, to write the solution in dimensional variables it is sufficient to multiply them by the corresponding normalizing factors by means of the formulas

$$
\begin{aligned}
& q_1^*(x_1) = q_1(x_1)d_3, \quad q_2^*(x_1) = q_2(x_1)d_3d_1^{-1}h \\
& d_1 = 1 \ \text{J/C}, \qquad d_2 = 10^{11} \ \text{V/m}, \\
& d_3 = 10^{11} \ \text{N/m}^2, \qquad h = 1 \ \text{m}
\end{aligned}
$$

The expressions for the given amplitude functions of the displacement and the potential have the form $f_1^*(x_1) = f_1(x_1)h$, $f_2^* = f_2 d_2 h$.

4. On the basis of the solutions obtained that describe the displacement and potential wave fields in the domain behind the electrode, we present a method of computing the values of the deviation of the excited wave phase velocity from the Rayleigh wave velocity at any given point of the piezocrystal surface.



The wave process in the domain behind the electrode is described by the wave packet

$$\theta(x_1) = \sum_{j=1}^{n} A_j(x_1) \exp(i\eta_j x_1)$$

which can be approximated by the wave

$$\theta(x_1) \sim C(x_1) \exp(i\varkappa_1 x_1)$$
$$\varkappa_1 x_1 = \mu_1 x_1 + m\pi, \ \mu_1 x_1 = \operatorname{arctg}(\operatorname{Re}\theta(x_1)/\operatorname{Im}\theta(x_1)), \ |\mu_1 x_1| < \pi$$

The wave number corresponding to the Rayleigh wave is known. We denote it by $\varkappa(\varkappa = \zeta)$, then $\varkappa x_1 = \mu x_1 + k\pi$. At infinity, at far zone points, the phase velocity of the excited wave agrees with the Rayleigh wave velocity, and therefore, the value of the argument of the function $\exp(i\varkappa_1 x_1)$ should be selected from the condition $m = k$.

Let Δv be the absolute value of the deviation of the excited wave phase velocity v_1 from the Rayleigh wave velocity v_0: $\Delta v = 10^3 (v_1 - v_0)/v_0$.

The table shows values of Δv at given points of the surface for the case of electrical $(f_1 = 0, f_2 = 1)$ and mechanical $(f_1 = 1, f_2 = 0)$ perturbations as well as for the superposition of both kinds of loads $(f_1 = 1, f_2 = 1)$ for electrodes with flat undersides $(\eta = 0)$ of width 1.5 λ.

f_1	f_2	$x_1/\lambda = 6.1$	10.1	20.1
0	1	0.27	0.16	0.08
1	0	0.49	0.29	0.14
1	1	0.50	0.29	0.14

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